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Higher Dimensional LRS Bianchi Type-I Domain Walls in a Scalar-Tensor Theory of Gravitation

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Abstract An exact higher dimensional LRS Bianchi type-I cosmological model is obtained in presence of thick domain walls in a scalar tensor theory of gravitation proposed by Saez and Ballester (Phys. Lett. A113:467, 1985). Some physical and kinematical properties of the models are also discussed.

Keywords Higher dimensions · Saez-Ballester's scalar-tensor theory · Domain walls · LRS Bianchi type-I model

1 Introduction

The study of higher dimensional cosmology space-time because of the underlying that the cosmos at its early stage of evolution of the universe might have had a higher dimensional era. The dimensionality of the world has long been a subject of discussion due to fact that our sense perceived only four dimensions, but there is nothing in the equation of relativity which restricts them to four dimensions. Witten [1], Applequist et al. [2], Chudos and Detweiler [3] and Marciano [4] are some of the authors who have initiated the discussion of higher dimensional cosmological models. A number of authors [5–7] have studied physics of the universe in higher-dimensional space-time. These models are believed to be physical relevance possibly at the early times before the universe has undergone compactification transitions.

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In the last few decades, there has been considerable interest in studying alternative theories of gravitation, the most important among them being scalar-tensor theories proposed by Lyra [8], Brans and Dicke [9], Nordtvedt [10] and Wagoner [11]. Subsequently, Saez and Ballester [12] have developed a new scalar-tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of weak fields. In spite of the dimensionless character of the scalar field an antigravity regime appears. This theory suggests a possible way to solve the missing matter problem in non-flat FRW cosmologies.

Phase transitions in the early universe can give rise to various forms of topological defects. These defects could be domain walls, cosmic strings, monopoles and textures. In particular, domain walls have become more important in recent years from cosmological standpoint in view of a new scenario of galaxy formation has been proposed by Hill et al. [13]. According to them the formations of galaxies are due to domain walls produced during phase transitions after recombination of matter and radiation. Vilenkin [14], Isper and Sikivie [15], Widrow [16], Goetz [17], Mukherjee [18], Wang [19], Rahaman et al. [20], Reddy and Rao [21] are some of the authors who have investigated several aspects of domain walls.

The field equations in the scalar tensor theory proposed by Saez and Ballester [12] are

$$R_{ij} - \frac{1}{2}Rg_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = T_{ij} \tag{1}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n\phi^{i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{2}$$

where n is an arbitrary constant, ω is a dimensionless coupling constant and other symbols have their usual meaning. Here comma (,) and semicolon (;) denote partial and covariant differentiation respectively.

Also the energy conservation equation

$$T^{ij}_{;j} = 0 \tag{3}$$

is a consequence of the field equations.

Rahaman and Bera [22], Chakraborty et al. [23] are some of the authors who have investigated domain walls in alternative theories of gravitation in four and five dimensions. Recently, Adhav et al. [24] discussed four dimensional non-static domain walls in Brans and Dicke [9] and Saez and Ballester [12] scalar-tensor theories of gravitation. While Reddy et al. [25] , exhibited several aspects of five dimensional domain walls in Saez and Ballester theory of gravitation. We hold the view that the investigation is not yet complete and there is a scope of future work which may unravel some of the hidden secrets of the universe.

The present work is the extension of Katore [27] in Saez and Ballester scalar tensor theory of gravitation. In this paper, the field equations for spatially homogeneous and anisotropic LRS Bianchi type-I model are solved in the frame work of Saez and Ballester [12] scalar tensor theory of gravitation when the source of energy momentum tensor is domain walls. To get a determinate solution, we have also used a relation between metric potentials and an equation of state corresponding to pressure and energy density of the thick domain walls.

2 Field Equations

We consider a homogeneous LRS Bianchi type I metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2) + C^2dw^2, \tag{4}$$

where A, B, C, are the function of t only.

A thick domain wall can be viewed as Solition like solution of the scalar field equations coupled with gravity. There are two ways of studying thick domain walls. One way is to solve gravitational field equations with an energy momentum tensor describing a scalar field ψ with self-interactions contained in a potential $v(\psi)$ given by

$$\psi_i\psi_j - g_{ij}\left[\frac{1}{2}\psi_k\psi^k - v(\psi)\right]. \tag{5}$$

Second approach is to assume the energy momentum tensor in the form

$$T_{ij} = \rho(g_{ij} + \omega_i\omega_j) + p_1\omega_i\omega_j, \omega_i\omega_j = -1, \tag{6}$$

where ρ is the energy density of the walls, p_1 is the pressure in the direction normal to the plane of the wall and ω_i is a unit space-like vector in the same direction.

Here we use the second approach to study the thick domain walls in Saez-Ballester theory.

In co-moving co-ordinate system from (6), we have

$$T_1^1 = T_2^2 = T_3^3 = T_4^4 = \rho, \qquad T_5^5 = -p_1 \quad \text{and} \quad T_j^i = 0 \quad \text{for } i \neq j. \tag{7}$$

The quantities ρ and p_1 depends on t only.

The field equations (1), (2) for the metric (4) with the help of (5), (6) and (7) can be written as

$$2\frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{B_5^2}{B^2} + 2\frac{B_5}{B}\frac{C_5}{C} - \frac{\omega}{2}\phi^n\phi_5^2 = \rho, \tag{8}$$

$$\frac{A_{55}}{A} + \frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{A_5}{A}\frac{B_5}{B} + \frac{A_5}{A}\frac{C_5}{C} + \frac{B_5}{B}\frac{C_5}{C} - \frac{\omega}{2}\phi^n\phi_5^2 = \rho, \tag{9}$$

$$\frac{A_{55}}{A} + 2\frac{B_{55}}{B} + \frac{B_5^2}{B^2} + 2\frac{A_5}{A}\frac{B_5}{B} - \frac{\omega}{2}\phi^n\phi_5^2 = \rho, \tag{10}$$

$$2\frac{A_5}{A}\frac{B_5}{B} + \frac{B_5^2}{B^2} + 2\frac{B_5}{B}\frac{C_5}{C} + \frac{A_5}{A}\frac{C_5}{C} + \frac{\omega}{2}\phi^n\phi_5^2 = -p_1, \tag{11}$$

$$\left(\frac{A_5}{A} + 2\frac{B_5}{B} + \frac{C_5}{C}\right)\phi_5 + \phi_{55} + \frac{n}{2}\frac{\phi_5^2}{\phi} = 0. \tag{12}$$

Equation (3) which is a consequence of the field equation takes the forms.

$$p_{1_5} + \left(\frac{A_5}{A} + 2\frac{B_5}{B} + \frac{C_5}{C}\right)(p_1 + \rho) = 0, \tag{13}$$

where the subscript 5 denote ordinary differentiation with respect to t.

3 Solution of the field equations

The five equations (8)–(12) contain six unknowns A, B, C, ϕ, ρ, p_1 and hence the system is indeterminate as it stands. We can introduce more conditions: either by an ad hoc assumption corresponding to some physical situation or an arbitrary mathematical supposition. However, both the procedures have some drawbacks: physical situation may lead to differential equations which will be difficult to integrate and mathematical supposition may lead to a non physical situation.

For complete determinacy of the system, one extra condition is needed. For this purpose, we assume $A = BC$.

Using above condition, the set of field equations (8)–(13) reduces to

$$2\frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{B_5^2}{B^2} + 2\frac{B_5}{B}\frac{C_5}{C} - \frac{\omega}{2}\phi^n\phi_5^2 = \rho, \tag{14}$$

$$2\frac{B_{55}}{B} + 2\frac{C_{55}}{C} + \frac{B_5^2}{B^2} + \frac{C_5^2}{C^2} + 5\frac{B_5}{B}\frac{C_5}{C} - \frac{\omega}{2}\phi^n\phi_5^2 = \rho, \tag{15}$$

$$3\frac{B_{55}}{B} + \frac{C_{55}}{C} + 3\frac{B_5^2}{B^2} + 4\frac{C_5}{C}\frac{B_5}{B} - \frac{\omega}{2}\phi^n\phi_5^2 = \rho, \tag{16}$$

$$3\frac{B_5^2}{B^2} + \frac{C_5^2}{C^2} + 5\frac{B_5}{B}\frac{C_5}{C} + \frac{\omega}{2}\phi^n\phi_5^2 = -p_1, \tag{17}$$

$$\left(3\frac{B_5}{B} + 2\frac{C_5}{C}\right)\phi_5 + \phi_{55} + \frac{n}{2}\frac{\phi_5^2}{\phi} = 0, \tag{18}$$

$$p_{15} + \left(3\frac{B_5}{B} + 2\frac{C_5}{C}\right)(p_1 + \rho) = 0. \tag{19}$$

We solve the above set of field equations (14)–(19) with the transformation

$$B = e^{\alpha},\ C = e^{\beta},\ dt = ABCdT = B^2C^2dT.$$

The above set of equation reduces to

$$2\alpha'' + \beta'' - \alpha'^2 - \beta'^2 - 4\alpha'\beta' + \frac{\omega}{2}\phi^n\phi'^2 = \rho e^{(4\alpha+4\beta)}, \tag{20}$$

$$2\alpha'' + 2\beta'' - \alpha'^2 - \beta'^2 - 3\alpha'\beta' - \frac{\omega}{2}\phi^n\phi'^2 = \rho e^{(4\alpha+4\beta)}, \tag{21}$$

$$3\alpha'' + \beta'' - \beta'^2 - 4\alpha'\beta' - \frac{\omega}{2}\phi^n\phi'^2 = \rho e^{(4\alpha+4\beta)}, \tag{22}$$

$$3\alpha'^2 + \beta'^2 + 5\alpha'\beta' + \frac{\omega}{2}\phi^n\phi'^2 = -p_1 e^{(4\alpha+4\beta)}, \tag{23}$$

$$\phi'' + \alpha'\phi' + \frac{n}{2}\frac{\phi'^2}{\phi} = 0, \tag{24}$$

$$p_1' + (3\alpha' + 2\beta')(p_1 + \rho) = 0, \tag{25}$$

where a superscript prime ($'$) indicates differentiation with respect to T.

For solving the above set of highly non-linear field equations, one has to assume physical or mathematical conditions. Here we assume [26]

$$\rho = p_1. \tag{26}$$

This condition is analogous to the stiff fluid (self-gravitating fluid) equation of state i.e. $\rho = p$ in general relativity.

Using (26), set of equations (20)–(25) admit an exact solution

$$A = \lambda_4(\lambda_1 T + \lambda_2)^{1+\lambda_3/\lambda_1},$$
$$B = (\lambda_1 T + \lambda_2),$$
$$C = \lambda_4(\lambda_1 T + \lambda_2)^{\lambda_3/\lambda_1}, \tag{27}$$
$$\phi = \left[\left(\frac{n+2}{2}\right)\log[\lambda_5(\lambda_1 T + \lambda_2)^{\lambda_3/\lambda_1}]\right]^{\frac{2}{n+2}}, \tag{28}$$
$$p_1 = \rho = \frac{\lambda_7}{(\lambda_1 T + \lambda_2)^{\lambda_6/\lambda_1}},$$

where λ_1, λ_2, λ_3, λ_4, λ_5, λ_7 are constants of integration and $\lambda_6 = 6\lambda_1 + 4\lambda_3$. After a proper choice of co-ordinates and constants, the metric (4) in a scalar-tenser theory of gravitation proposed by Saez and Ballester (1985) becomes

$$ds^2 = -d\tau^2 + \tau^{2\mu} dX^2 + \tau^2(dY^2 + dZ^2) + \tau^2 dW^2. \tag{29}$$

4 Some Physical and Kinematical Properties

Equation (29) represents an exact five dimensional LRS Bianchi type-I domain walls in the frame work of scalar tensor theory of gravitation, proposed by Saez and Ballester [12]. It is interesting to note that the model is free from singularities. For $\mu = 1$, the model becomes isotropic.

For the model (29), the physical and kinematical variables which are important in cosmology are

$$\text{Spatial volume} = \sqrt{-g}$$
$$= \lambda_4^2 \tau^{\lambda_8}, \quad \text{where } \lambda_8 = 3 + 2\frac{\lambda_3}{\lambda_1}. \tag{30}$$

Scalar expansion $\theta = u^i_{;i}$

$$\theta = \frac{\lambda_9}{\tau}, \quad \text{where } \lambda_9 = \left(1 + 2\lambda_1 + 2\frac{\lambda_3}{\lambda_1}\right). \tag{31}$$

Shear scalar:

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{4\lambda_9^2}{9\tau^2}. \tag{32}$$

Hubble parameter:

$$H = \frac{R_4}{R} = \frac{1}{3\lambda_4^2 \tau^{\lambda_{10}}}, \quad \text{where } \lambda_{10} = 3 + \frac{2\lambda_3}{\lambda_1}. \tag{33}$$

The spatial volume increases with τ and it becomes infinite for large values of τ. Thus inflation is possible for large τ. Also Volume becomes zero at the instant $\tau = 0$ and hence there is a Big-Bang at $\tau = 0$, $\rho \to 0$ as $\tau \to \infty$. Thus the model gives essentially an empty universe as $\tau \to \infty$.

It can be observed that for large τ, the parameters θ, σ, H vanish and diverge when $\tau \to 0$.

Also for $\tau = 0$, the scalar field ϕ diverges.

Also for large value of τ, the ratio $(\frac{\sigma^2}{\theta^2}) \neq 0$ and hence the model (29) does not approach isotropy.

5 Conclusion

It is observed that the model (29) is non-singular in five dimensions, which is similar to that of [27] in four-dimensions. The model is inflationary and does not approach isotropy. The model obtained can be viewed as a five dimensional self-gravitating or stiff domain wall in this theory. In view of the fact that there is an increasing interest in recent years, in scalar fields in general relativity and alternative theories of gravitation in the context of inflationary cosmology, this model obtained has considerable astrophysical significance. It is observed that the model (29) is quite similar to five dimensional string model [26] in the Saez-Ballester theory of gravitation. This model will help for a better understanding of the structure formation at the early stages of Evolution of the universe.

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